When comparing two positive numbers, the larger number always has the larger square.

EXAMPLE | Is $\sqrt{10}$ more or less than 3.5?

Since $\sqrt{10}$ and 3.5 are both positive, we can compare $\sqrt{10}$ and 3.5 by comparing their squares.

$\sqrt{10}$ is the number we square to get 10. So, $\left(\sqrt{10}\right)^2 = 10$.

We have $3.5^2 = 12.25$.

Since $10 < 12.25$, we know $\sqrt{10}$ is **less than** 3.5.

PRACTICE | Fill each circle below with < or > to indicate which expression is greater.

51. $2 \bigcirc \sqrt{5}$

52. $\sqrt{3{,}599} \bigcirc 60$

53. $200 \bigcirc \sqrt{36{,}000}$

54. $\sqrt{250} \bigcirc 16$

55. $50 \bigcirc \sqrt{49 \cdot 51}$

56. $111 \bigcirc \sqrt{12{,}345}$

57. $\sqrt{\frac{1}{2}} \bigcirc \frac{2}{3}$

58. $\frac{7}{8} \bigcirc \sqrt{\frac{3}{4}}$

PRACTICE | Answer each question below.

59. For how many integer values of a is $\sqrt{\frac{1}{a}} > \frac{1}{2}$?

59. ________

60. ★ Is $\sqrt{250}$ closer to $\sqrt{100}$ or to $\sqrt{400}$?

60. ________

61. ★ Round $\sqrt{30.3}$ to the nearest whole number.

61. ________

EXAMPLE Which is greater, $2\sqrt{3}$ or $3\sqrt{2}$?

The expression $2\sqrt{3}$ means $2\cdot\sqrt{3}$.

To compare $2\sqrt{3}$ to $3\sqrt{2}$, we can compare their squares.

$(2\sqrt{3})^2$	$(3\sqrt{2})^2$
$=2\cdot\sqrt{3}\cdot 2\cdot\sqrt{3}$	$=3\cdot\sqrt{2}\cdot 3\cdot\sqrt{2}$
$=(2\cdot 2)\cdot(\sqrt{3}\cdot\sqrt{3})$	$=(3\cdot 3)\cdot(\sqrt{2}\cdot\sqrt{2})$
$=4\cdot 3$	$=9\cdot 2$
$=12$	$=18$

Since 18 is greater than 12,
$\mathbf{3\sqrt{2}}$ is greater than $2\sqrt{3}$.

PRACTICE Answer each question below.

62. What is the area in square centimeters of a square whose sides are $4\sqrt{5}$ centimeters long? **62.** ________

63. Circle every expression below that equals $3\sqrt{8}$.

$\sqrt{72}$ $2\sqrt{18}$ $4\sqrt{6}$ $5\sqrt{3}$ $6\sqrt{2}$ $\frac{17}{2}$

64. Circle every expression below that equals $\sqrt{500}$.

$25\sqrt{2}$ $20\sqrt{5}$ $10\sqrt{5}$ $8\sqrt{15}$ $5\sqrt{20}$ $2\sqrt{125}$

65. Order 11, $\frac{\sqrt{500}}{2}$, and $2\sqrt{30}$ from least to greatest. **65.** _______<_______<_______

66. ★ If n is an integer, and $13 < n\sqrt{11} < 14$, then what is n? **66.** n = ______

EXAMPLE | Compute $\sqrt{25^3}$.

We could compute $25^3 = 25 \cdot 25 \cdot 25$ and then look for the square root of the result.

Or, we can use the fact that $25 = 5 \cdot 5$ to help us find the square root of 25^3.

$$\begin{aligned}\sqrt{25^3} &= \sqrt{25 \cdot 25 \cdot 25}\\ &= \sqrt{(5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5)}\\ &= \sqrt{(5 \cdot 5 \cdot 5) \cdot (5 \cdot 5 \cdot 5)}\\ &= \sqrt{(5 \cdot 5 \cdot 5)^2}\\ &= 5 \cdot 5 \cdot 5\\ &= 125\end{aligned}$$

So, the square root of 25^3 is **125**.

PRACTICE | Write each expression below as an integer.

67. $\sqrt{11^4}$ = __________

68. $\sqrt{7^6}$ = __________

69. $\sqrt{9^3}$ = __________

70. $\sqrt{4^5}$ = __________

71. $\sqrt{3^2 \cdot 2^8}$ = __________

72. $\sqrt{3^4 \cdot 5^2}$ = __________

73. $\sqrt{6^2 \cdot 15^2}$ = __________

74. $\sqrt{12^3 \cdot 3}$ = __________

75. ★ If $\sqrt{2^n} = 64$, then what is n?

75. n = ______

76. ★ What is the units digit of $\sqrt{3^{100}}$?

76. __________

EXAMPLE | Compute $\sqrt{6 \cdot 24}$.

We multiply $6 \cdot 24 = 144$, then find the square root.

$$\begin{aligned}\sqrt{6 \cdot 24} &= \sqrt{144} \\ &= \mathbf{12}.\end{aligned}$$

— ***or*** —

Using prime factorization, we have

$$\begin{aligned}\sqrt{6 \cdot 24} &= \sqrt{(2 \cdot 3) \cdot (2 \cdot 2 \cdot 2 \cdot 3)} \\ &= \sqrt{(2 \cdot 2 \cdot 3) \cdot (2 \cdot 2 \cdot 3)} \\ &= \sqrt{(2 \cdot 2 \cdot 3)^2} \\ &= 2 \cdot 2 \cdot 3 \\ &= \mathbf{12}.\end{aligned}$$

PRACTICE | Solve each problem below.

77. $\sqrt{4 \cdot 9} =$ __________

78. $\sqrt{3 \cdot 27} =$ __________

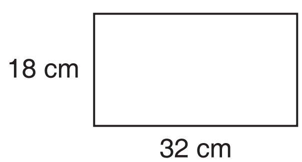
79. $\sqrt{32 \cdot 8} =$ __________

80. $\sqrt{21 \cdot 84} =$ __________

81. $\sqrt{135 \cdot 15} =$ __________

82. $\sqrt{2{,}916} =$ __________

83. What is the side length in centimeters of a square that has the same area as the rectangle below?



83. ____________

84. The prime factorization of 1,382,976 is $2^6 \cdot 3^2 \cdot 7^4$. What is the prime factorization of $\sqrt{1{,}382{,}976}$?

84. ____________

85. ★ The expression $\sqrt{540 \cdot k}$ is equal to an integer for some positive integer k. What is the smallest possible value of k?

85. ____________