Beast Academy 1
Chapter 1: Counting

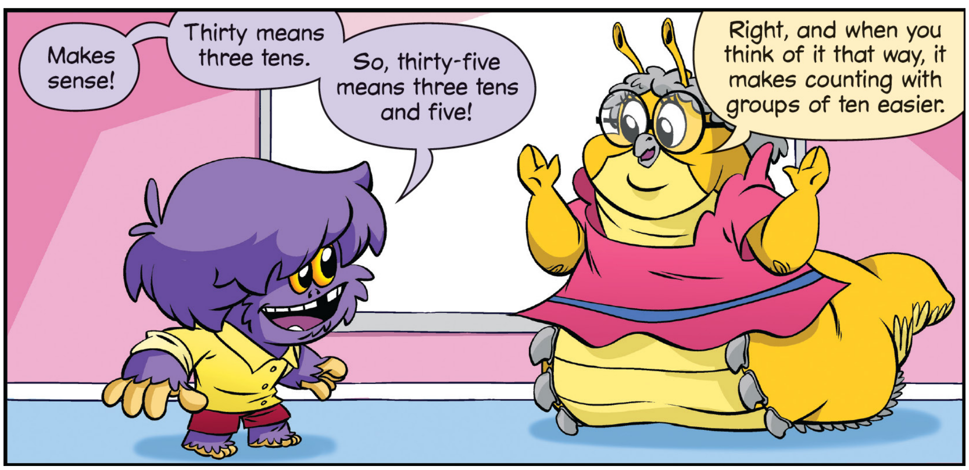
Sequence: BA1, Chapter 1 This Chapter ▸ BA1, Chapter 3 Comparing ▸ BA1, Chapter 4 Addition ▸ BA1, Chapter 5 Subtraction •••

Students beginning BA1 must be able to count fluently to at least 20, and should know how to read numbers written in digits up to 100.

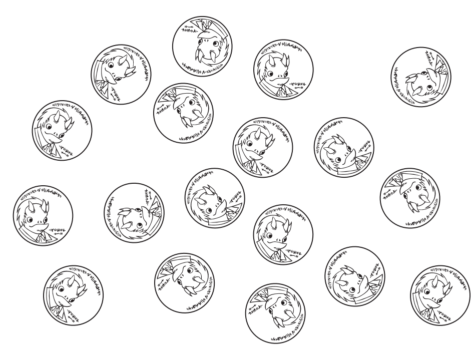
Overview

In this chapter, students work on one-to-one correspondence, gain practice recognizing small amounts (subitization), and gain fluency with numbers to 100. **The first half of this chapter should be a review of foundational counting skills for most students.**

Then, we lay the groundwork for place value by focusing on the number ten.



One-to-one Correspondence



We begin this chapter with a review of foundational skills that students should already have some familiarity with.

Even young students who count fluently often have difficulty counting items that are not organized (it is much easier to count pennies in rows than if they are scattered). Students often miss a penny or count the same penny more than once. Students may even count the same set twice and get different results without recognizing a contradiction.

For these students, counting is like saying the alphabet. The numbers don't yet have meaning. The early practice in Chapter 1 helps students see that numbers stand for quantities. When counting, students must assign one number to each object for their final count to be correct.

Beast Academy 1
Chapter 1: Counting

Subitizing

Subitizing means recognizing a number of items at a glance without counting. For example, you can probably quickly recognize the number of dots on any face of a standard die.

Students who practice seeing small numbers without counting gain intuition for many important number facts and properties and develop number sense. For example, recognizing that one more than 4 is 5, or that two 3's always make 6. Students may even start to recognize properties of numbers like odds and evens on their own.

You may quickly recognize that each of these pairs of dice shows 9 dots.

Consider all of the things a student can learn about 9 using just these two pairs!

5+4=9	6+3=9	5+4=6+3
4+4+1=9	5+2+2=9	3+3+3=9

Ten

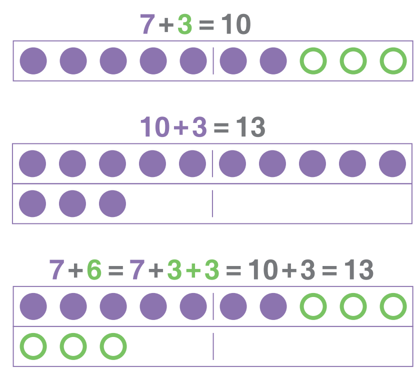
Our entire number system is built around the number ten. So, we spend a lot of time focusing on the number ten in Beast Academy.

Using ten frames and ten strips is a good way for students to see relationships between numbers and ten.

For example, students should begin to recognize pairs of numbers that sum to ten (6+4, 7+3, and so on).

Students should also see that numbers from 10 to 19 have 1 ten and some extras, and that numbers in the 50's have 5 tens and some extras.

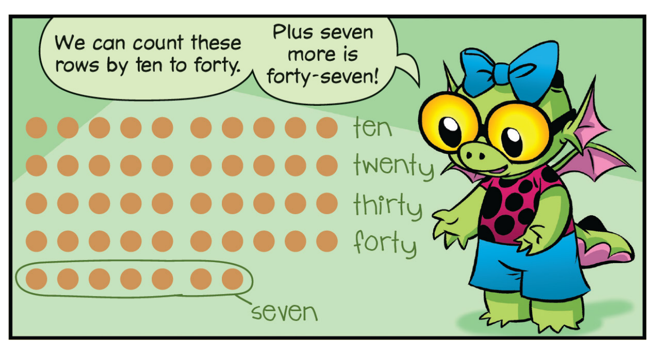
Soon, students will begin to use these relationships to add pairs of numbers with strategies that involve making ten.



To 100

Next, students get a solid understanding of numbers within 100 with practice:

- Starting the counting sequence starting at any number within 100.
- Counting by tens to find quantities of items arranged in groups of ten.
- Recognizing that the left digit in a 2-digit number gives the number of tens it has.



Students should be recognizing patterns in the digits of a hundreds chart and making connections between visual models and numbers. We're building towards an understanding of place value.

We don't use terms like "digits" or the "tens place" and the "ones place" yet, but students are beginning to think about two-digit numbers as groups of tens and ones by the end of the chapter.

Beast Academy 1
Chapter 2: Shapes

AoPS Beast Academy
Art of Problem Solving

Sequence: BA1, Chapter 2 This Chapter → BA1, Chapter 6 Categories → BA3, Chapter 1 Shapes → BA4, Chapter 1 Shapes • • •

Students should be familiar with the names of basic shapes like triangles, circles, rectangles, and squares.

Overview

In this chapter, students learn the names and properties of basic shapes and get practice drawing them. They also learn to recognize spins (rotations) and flips (reflections) of a shape. Finally, students learn to split and combine shapes to make other shapes.

We do not include two grade-1 Common Core geometry standards.

These include composing and decomposing 3D shapes as well as partitioning circles and rectangles into quarters and halves.

Why do we skip these standards?

It's hard to explain 3D solids on two-dimensional workbook pages. The 3D standards are better accomplished with actual blocks that students can pick up and play with. We encourage students play with 3D solids to form other shapes wherever possible.

We introduce fractions much later (BA3, Chapter 10) as numbers on the number line. Introducing fractions as parts of shapes can make it more difficult for students to understand fractions as numbers later on.

Shapes

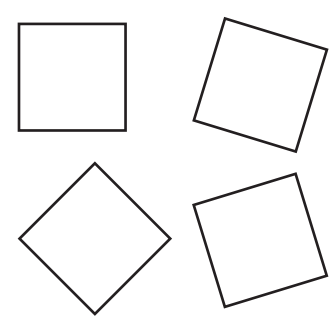
We give a very basic introduction to several shapes.

Students practice drawing shapes to familiarize themselves with basic shapes and their properties before attempting the more challenging sections that follow.

We also encourage students to see that shapes are categorized based on properties like number of sides and corners, not orientation. For example, a square is still a square, no matter which way you turn it.

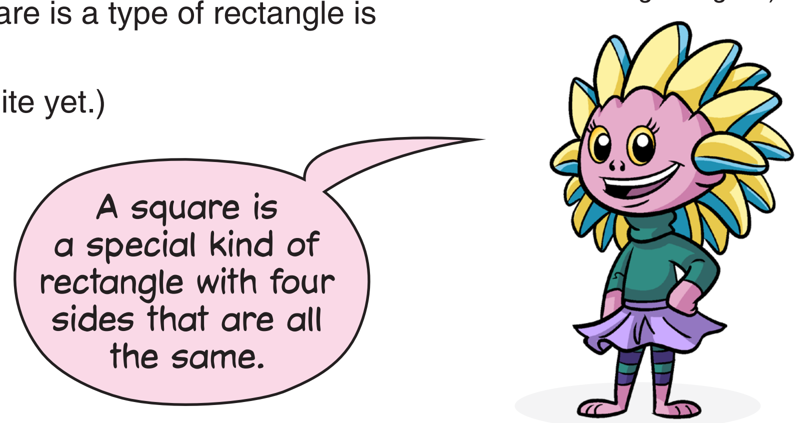
Finally, students begin to see how shapes are classified. For example, most students recognize that rectangles are a type of shape. But, the idea that a square is a type of rectangle is tough for students at this age.

(It's fine if they don't grasp this quite yet.)



All of these are squares.

And, they are all rectangles! (They all have four sides and four right angles.)



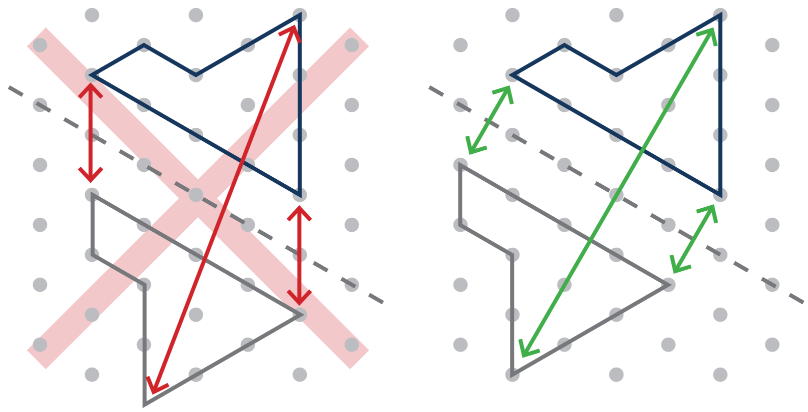
Beast Academy 1
Chapter 2: Shapes

Reflections and Rotations

Many geometry problems are made easier by imagining a shape in a different orientation.

Flips tend to be easier for students across a straight horizontal or vertical line. Each point corresponds to one that is “straight across” from it on the other side of the line. Students are using perpendicular lines without thinking about it! Students should practice reflecting across diagonal lines without turning their books, but it’s fine to turn their books to check.

It’s also a good idea for students to physically manipulate shapes like the cut out figures in the book. Using their hands to flip and turn actual shapes helps students make connections that are harder to make with pictures on a page.



Students may draw the right shape but in the wrong place, as shown above on the left. Every point must be reflected “straight across” (perpendicular to) the mirror line as shown in the correct answer on the right.

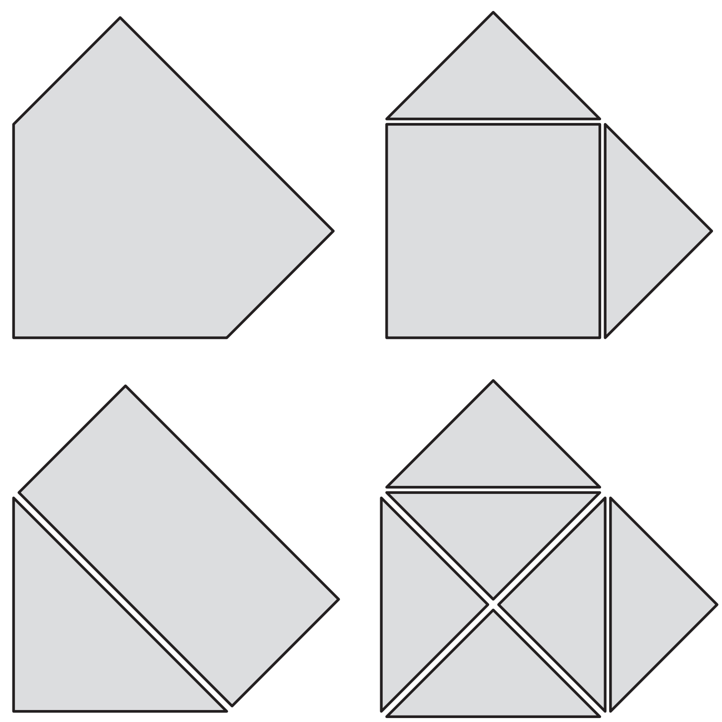
Splitting and Combining Shapes

Many geometry problems are made easier by splitting or combining shapes into ones that are easier to work with.

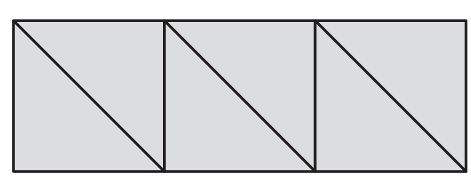
For example, a difficult area problem might be made easier by splitting a shape into smaller triangles, rectangles, or other familiar shapes.

Students practice splitting shapes into rectangles, triangles, and squares, and learn that shapes can be split in more than one way.

Finally, students get their first taste of polyominoes (shapes made by attaching squares along their edges) and find ways that polyominoes can be combined into other shapes.



There are many ways to split the shape above into smaller shapes. We can then rearrange and combine these shapes in other ways. For example, we can rearrange the six small triangles above to make the rectangle below.



Beast Academy 1
Chapter 3: Comparing

Sequence: BA1, Chapter 3 This Chapter → BA1, Chapter 8 Comparing → BA2, Chapter 2 Comparing

Students should be able to count fluently to 100 and have a basic understanding of place value from Chapter 1.

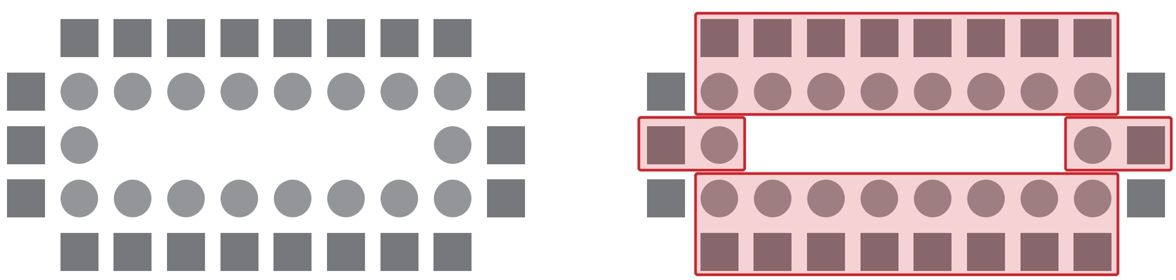
Overview

We begin the chapter by helping students understand that when comparing two amounts, we can ignore the parts that are equal. We then move on to comparing and ordering amounts using the symbols for less than (<), greater than (>), and equals (=). We finish by comparing lengths and distances without measuring.

Equal Amounts

Sometimes, the best way to compare amounts is to start by finding the parts that are equal. If we remove the parts that are equal, we can compare what's left.

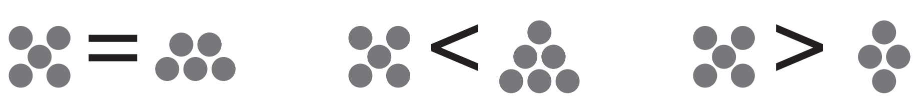
For example, if we want to know whether there are more squares or more circles below, we can remove equal numbers of squares and circles to see that there are four squares left. So, there are more squares than circles. (There are lots of other ways to find equal groups.)



We use this idea throughout the curriculum, often without thinking about it. For example, when comparing large numbers later on, students discover that they can ignore all of the parts that are equal. 65,43<u>2</u> is greater than 65,43<u>1</u>.

Symbols

We use symbols to compare amounts. Learning these symbols gives students a chance to start reading and writing formal math notation and thinking abstractly (with symbols, rather than objects). Most students learn that the < and > symbols "eat" the bigger number. The = symbol means that two amounts are equal.



$5 = 5$	$5 < 6$	$5 > 4$
"Five equals five."	"Five is less than six."	"Five is greater than four."

Beast Academy 1
Chapter 3: Comparing

Ordering

To compare more than two numbers, it makes sense to look at the larger place values first. Try not to teach this as a rule or a trick. Students can see why just by looking at the counting sequence. For example, any number in the fifties is larger than any number in the forties.

So, when ordering numbers in the forties and fifties, we separate the 40's from the 50's, then arrange both groups in order. Later, students will use the same principles to order lists of numbers that have more digits.

Order the following numbers from smallest to largest:

48 51 43 55 54 44 40

First, we separate the 40's from the 50's.

48 43 44 40 51 55 54

Then, we put each set in order.

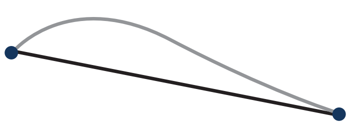
40 43 44 48 51 54 55

Comparing Lengths

We compare lengths in this chapter without measuring (that will come in chapter 11). Students can apply the same concepts from earlier in the chapter, ignoring parts that are equal.

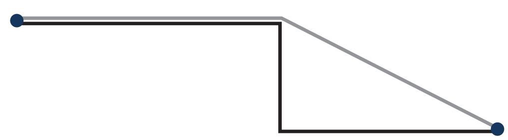
Or, we can compare corresponding parts, as shown in the example on the right.

Which path is longer, gray or black?



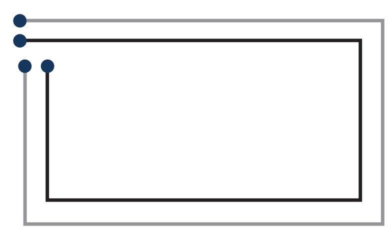
A straight path gives the shortest distance between two points. So, the gray path is longer.

Which path is longer, gray or black?



The left sides of both paths are the same length. We can ignore those parts. The rest of the black path is longer. So, the black path is longer.

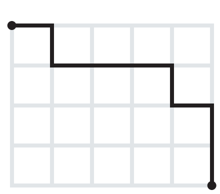
Which path is longer, gray or black?

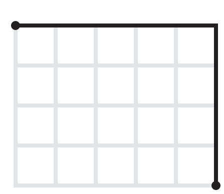


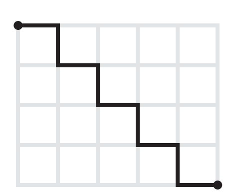
Each side of the gray path is a little longer than the same side of the black path. So, the gray path is longer.

City Walks

It may be surprising for students that all of the paths below are all the same length. Once again, we look for parts that are equal that we can ignore. Starting in the top-left corner, each walk includes a total of 5 blocks right and 4 blocks down. So, all of the paths are 9 blocks long.







Beast Academy 1
Chapter 4: Addition

AoPS Beast Academy
Art of Problem Solving

Sequence: ••• BA1, Chapter 3 Comparing ▸ BA1, Chapter 4 This Chapter ▸ BA1, Chapter 5 Subtraction ▸ BA1, Chapter 7 Addition & Subtrac... •••

Students should be fluent counting within 100 starting at any number.

Overview

In this chapter, students review the basic meaning of addition, as putting items of the same kind all together. They then begin working with addition equations, learning to read and interpret math symbols correctly as “math sentences”. Students are then introduced to several basic strategies that include using commutativity ($5+3=3+5$), compensation ($9+7=10+6$), and lots of strategies that involve making and adding ten.

Addition Basics

Addition is usually the first mathematical operation kids learn. Most students in Beast Academy Level 1 are probably already familiar with the basics, so the early pages of this chapter should be mostly review.

Students can add by counting the total number of items, but with practice, students should remember facts like doubles ($3+3=6$, $4+4=8$, etc.) and pairs that sum to 10 ($3+7$, $2+8$, etc.)

When introducing any new topic at this age, we rely on visual models for much of the early practice, then nudge students away from them as they understand the meaning of more abstract math expressions and equations.

It is not explicit in the chapter but worth noting that the items we add should be alike.

Addition Equations

Students often see addition presented as shown in the examples below for a long time.

$$5+3=\square \qquad 4+5=\square \qquad 7+5=\square$$

Students who only see equations this way may think the equals sign means, “What’s the answer?” This can lead to confusion later on when students learn to manipulate equations.

To help them understand the meaning of the equals sign, Beast Academy students are asked to fill in blanks like the ones in the equations below where the arrangements vary.

$$5+\square=8 \qquad \square=4+5 \qquad 8+6=\square+5$$

The equals sign lets us know that the values on both sides are the same.

This is an early introduction to algebraic thinking. The empty boxes above are like the variables in algebraic equations. Additionally, equations like $5+\square=8$ help students understand the relationship between addition and subtraction. $5+\square=8$ is a subtraction problem in disguise.

Beast Academy 1
Chapter 4: Addition

Strategies

Students learn several important properties and strategies that make solving many addition problems easier.

The Commutative Property of Addition

When adding two numbers, it doesn't matter what order they are in. (We don't call it the commutative property in the chapter, but you are welcome to tell your student that there is a long word in math that describes the fact that $1+2=2+1$.)

Students in BA Level 1 should recognize that rearranging a pair of dice doesn't change the number of dots. Very young students who don't have this concept, called "conservation of number," are not ready to begin adding.

$$5+4 \quad = \quad 4+5$$

Compensation

Compensation in addition means that you can move items from one pile to another without changing the total. This is especially useful later, where we can use the same strategy to change a problem like $97+55$ into $100+52$.

$$5+3 \quad = \quad 4+4$$

Ten

Students get lots of practice with 10. Students first practice recognizing pairs that sum to 10, and should begin to see the usefulness of these pairs. For example, when adding several small numbers, it is helpful to find a pair that adds to 10. (Since, for example, $10+3$ is easier than $8+5$.)

$$5+3+5=13 \qquad 5+2+8=15$$

$$10+3 \qquad 5+10$$

Students also learn that in every two-digit number, the left digit tells you how many tens the number has. They learn to add ten to any number within 100 (e.g. $8+10=18$ or $10+53=63$) and learn to add any digit to a number of tens like 30, 40, or 80 (e.g. $80+7=87$ or $4+70=74$).

Counting Up

While students are solidifying strategies like the ones above, it is perfectly fine if they are still counting up on their fingers to add small amounts, especially when crossing a multiple of ten (for example, to add $37+5$, where we cross 40 to get to 42). Even math beasts do this sometimes.

Beast Academy 1
Chapter 5: Subtraction

AoPS Beast Academy
Art of Problem Solving

Sequence: BA1, Chapter 4 Addition → **BA1, Chapter 5 This Chapter** → BA1, Chapter 7 Addition & Subtrac... → BA1, Chapter 8 Comparing

Students should be fluent adding small numbers within 100 and counting backwards within 100 starting at any number.

Overview

There are two important ways for students to think about subtraction.

Most students first learn that subtraction means taking away one amount from another. “If I have 9 apples and I give away 7 apples, how many apples will I have left?”

Subtraction is also used to find the difference between two amounts. “If I have 9 apples and you have 7 apples, how many more apples do I have?”

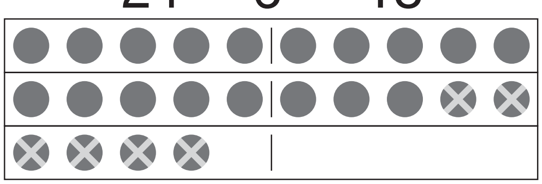
In this chapter, we introduce both ways as well as several important strategies.

Taking Away	Finding a Difference
What’s left if I take 7 from 9?	How much more is 9 than 7?
$9-7=\boxed{2}$	$9-7=\boxed{2}$
If I take 7 from 9, I have $\boxed{2}$ left.	9 is $\boxed{2}$ more than 7.

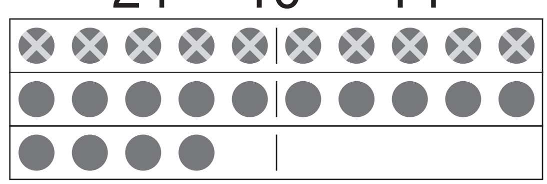
Taking Away

Most people first think of subtraction as taking away, and this is how subtraction is introduced in Beast Academy. We include ten frames in the early practice to encourage students to think about crossing ten in subtraction in two steps. For example, to subtract 24 − 6, students can subtract 4 first to get 20, then 2 more to get 18. Ten frames also help students see that subtracting 10 removes a whole row. So, 24 − 10 = 14.

$$24 - 6 = 18$$



$$24 - 10 = 14$$



Finding a Difference

The second use of subtraction is finding the difference between two numbers. For example, to find out how much older grandpa (72) is than grandma (67), we subtract 72 − 67. But we aren’t taking away 67 years from grandpa!

In problems like 72 − 67, it is easier to think about the difference, or “How much more is 72 than 67?” than it is to take away. But in a problem like 72 − 5, it is easier to take away.

It is important to be able to think of subtraction both ways.

Beast Academy 1
Chapter 5: Subtraction

Using Addition to Subtract

Subtraction can be used to find the missing number in an addition problem. For example, to solve a problem like $17+\square=21$, we can subtract $21-17$.

But, it is often useful to think about this the other way around. To answer $21-17$, we can ask, “What do I add to 17 to get 21?” For many, filling the blank in $17+\square=21$ is easier than $21-17=\square$.

$$21-17=\square$$

$$17+\square=21$$

Reordering Subtraction

When we’re subtracting more than one number, the order of the numbers we subtract doesn’t matter.

Reordering subtraction is not the same as reordering addition. 5+3 equals 3+5, but 5–3 does not equal 3–5.

Many teachers are understandably hesitant to tell students that they can rearrange subtraction. It can be scary! Be careful! But taught well, rearranging can make many subtraction problems much easier. It forces students to think carefully about how subtraction works and encourages strong number sense.

It is important for students to consider when and why reordering is allowed.

Students can recognize that taking away 2 then 3 is the same as taking away 3 then 2. In both cases, you take away a total of 5. Start with simple numbers and concrete examples like the grapes examples in the BA Guide section starting on page 126.

Like most math, this is not a concept that should be taught as a rule to memorize. Students will need to think carefully, asking themselves, “Does this make sense?” Even experienced students should consider concrete examples like taking away grapes. When in doubt, students can always work from left to right.

Subtract the easier number first:

$$\begin{aligned} &43-8-3\\ =\ &43-3-8\\ =\ &40-8\\ =\ &32 \end{aligned}$$

Subtract everything at once:

$$\begin{aligned} &43-7-3\\ =\ &43-10\\ =\ &33 \end{aligned}$$

Subtracting Part-by-Part

This is related to the concept above of subtracting everything at once, but instead of combining numbers being subtracted, we can split a number into parts that are easier to subtract.

For example, to subtract $45-9$, we can take away 5 first, then 4 more.

This is easier than trying to count down, or using the traditional method of stacking and ‘borrowing.’ We avoid stacking subtraction in BA until students have developed strong number sense and mental math skills that come with using strategies like these.

Subtracting one part at a time.

$$\begin{aligned} &45-9\\ =\ &45-5-4\\ =\ &40-4\\ =\ &36 \end{aligned}$$

Beast Academy 1
Chapter 6: Categories

Sequence: BA1, Chapter 2 Shapes → **BA1, Chapter 6 This Chapter** → BA3, Chapter 1 Shapes → BA4, Chapter 1 Shapes ...

Students should be familiar with the names of basic shapes like triangles, circles, rectangles, and squares.

Overview

Things get separated into categories in just about every subject area. In mathematics, we have lots of different categories for shapes and numbers.

In this chapter, students first look for properties that help them find differences in shapes.

Then, we explore several categories of numbers: odds and evens, perfect squares, and primes. Seeing these numbers early will serve as a foundation for more advanced work in later chapters.

Finally, we explore how Venn diagrams can be used to show how categories relate.

Similarities and Differences

Items can be categorized based on their similarities and differences. Students look for properties like color, shape, size, and quantity to describe how items are the same or different.

In the Odd One Out section, student often overlook the fact that all of the other items need to have something in common to belong to a group.

Odds and Evens

Students are often taught the rule that even numbers end in 0, 2, 4, 6, or 8, and odd numbers end in 1, 3, 5, 7, or 9 without understanding why these numbers are special.

There are two useful ways to think of even numbers:

- An even number can be split into two equal groups.
- An even number can be split into groups of two (pairs) with none left over.

If you try to split an odd number into two equal groups or groups of two, there will be one extra.

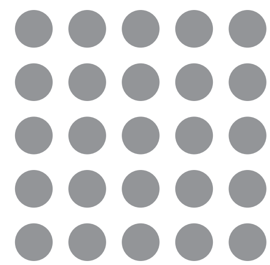
There are lots of problems (called parity problems) that can be solved just by looking at whether an amount is odd or even. We even dedicate all of Chapter 9 in Beast Academy 2C to odds and evens!

Beast Academy 1
Chapter 6: Categories

Perfect Squares

If we can arrange a set of dots in a grid with the same number in every row and column, the total number of dots is called a perfect square. For example, a 5-by-5 grid of dots has a total of 25 dots. So, 25 is a perfect square. The first few perfect squares are 0, 1, 4, 9, 16, 25, 36, and they continue forever.

Perfect squares appear throughout Beast Academy, and we even dedicate all of Chapter 5 in Beast Academy 3B to them!



25 dots can be arranged into a square, so 25 is a **perfect square** number.

Prime Numbers

A prime number has exactly two factors: 1 and itself.

But, we haven't taught students to multiply yet, and they probably don't know the term "factors," so we explain primes using shapes.

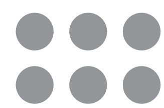
If, using a certain number of items, the only rectangles that can be made are 1 unit thick, we call the number **prime**.

Numbers that can make rectangles that are more than 1 unit thick are called **composite**.

The number 1 is special. One is not prime or composite. One is the building block for all other numbers!

The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, and they continue forever.

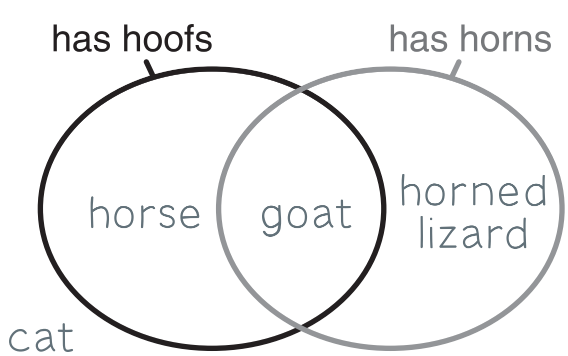
5 dots can't be arranged to make a rectangle with more than 1 row. So, 5 is **prime**.

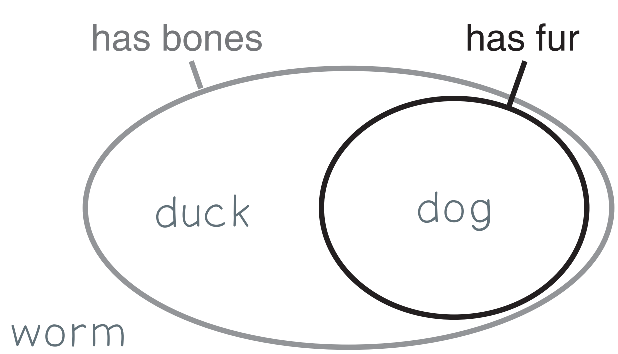


6 dots can be arranged to make a rectangle with more than 1 row. So, 6 is not prime. 6 is **composite**.

Venn Diagrams

Venn diagrams are a great way to show how categories overlap. Categories can overlap like the example on the left. Or, one category can be completely contained inside another category as shown on the right. Students get practice placing items where they belong in Venn diagrams like these.





Items that belong to both categories go in the area of overlap. Items that belong to neither go outside of both circles.

Beast Academy 1
Chapter 7: Addition & Subtraction

AoPS Beast Academy
Art of Problem Solving

Sequence: ••• BA1, Chapter 4 Addition ▸ BA1, Chapter 5 Subtraction ▸ BA1, Chapter 7 This Chapter ▸ BA1, Chapter 8 Comparing •••

Students should be familiar with addition and subtraction, including strategies in chapters 4 and 5: counting up, adding and subtracting ten, compensation, re-ordering, making ten, and using both models of subtraction (taking away and finding a difference). Students should also recognize every two-digit number as a number of tens and some extras (ones).

Overview

We begin with a review of some of the strategies learned in chapters 4 and 5.

Then we introduce students to the number line. The number line is an important model used throughout the BA curriculum, starting with basic addition and subtraction in this chapter, and later on with fractions, decimals, and negative numbers.

The second half of the chapter focuses on the importance of ten and furthers the introduction to place value that began in books 1A and 1B.

An important note about algorithms

We do not use traditional stacking algorithms like the one below yet. Instead, we encourage a variety of strategies that help students gain strong foundational number sense. This will help them compute quickly while also understanding what they are doing.

Students who are introduced to math as a set of processes to be memorized (like the stacking algorithm) often develop a just-tell-me-how-to-do-it mindset around math — wanting to know the formula or the trick or the steps they can use. Students who understand **why** what they are doing works recognize that they can figure most things out on their own.

Different problems can be solved different ways. Students should be flexible with the methods they use and can often develop strategies of their own.

Subtract 85 − 79.

Encourage This

"I know 85 is 6 more than 79, so 85 − 79 is 6."

$79 + \boxed{6} = 85$

$85 - 79 = \boxed{6}$

Or This

"I can count up to see 85 is 6 more than 79, so 85 − 79 is 6."

79 80 81 82 83 84 85

$85 - 79 = \boxed{6}$

Not This

"I stack the two numbers. Since 9 is bigger than 5, I need to make the 5 a 15, and take 1 away from the 8. Then, 15 − 9 is 6 and 7 − 7 is 0, but I don't write the 0 and the answer is 6."

$$\begin{array}{r} 85 \\ -\ 79 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 7\ 15} \\ \not{8}\not{5} \\ -\ 79 \\ \hline \end{array} \qquad \begin{array}{r} {\scriptstyle 7\ 15} \\ \not{8}\not{5} \\ -\ 79 \\ \hline 6 \end{array}$$

If a student is relying on the stacking algorithm, give problems where stacking is inefficient (like the one above) and encourage other methods. Students probably don't understand place value well enough yet to understand the "borrowing" step above. We save the traditional algorithm until BA level 2, after students have built a diverse set of tools and a strong understanding of place value.

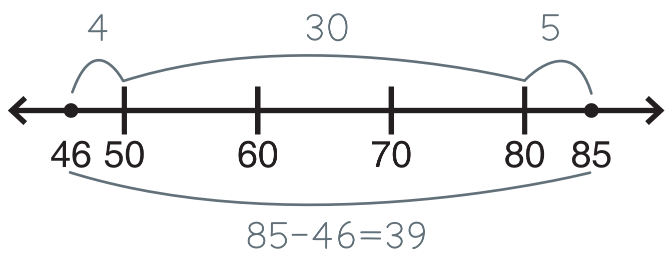
Beast Academy 1
Chapter 7: Addition & Subtraction

The Number Line

The number line gives students a way to visualize how numbers relate. Students can use the number line to compare numbers and as a model to help them with addition and subtraction. This works for whole numbers now, and is great for fractions and negative numbers later on.

To add, students can count the number of “hops” up (to the right) on the number line. To subtract, they can count “hops” down (to the left). Be sure students are counting hops, not tick marks.

With practice, students can make larger ‘hops’ across gaps, using multiples of ten as benchmarks. This is especially helpful when visualizing subtraction as the difference (or distance) between two numbers. For example, we can subtract 85 − 46 by finding the distance between 46 and 85 as shown.



Tens and Place Value

In this chapter, students begin adding by place value through a careful progression of steps that build intuition and understanding for why it works. Students first practice counting by tens, beginning at any number up to 100.

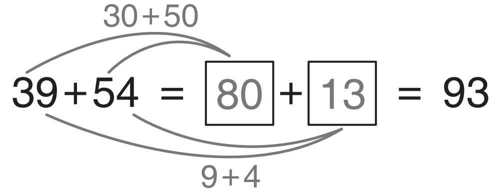
10 20 30 40 50   17 27 37 47 57

Students should recognize a two digit number as a number of tens and some extras (ones). So, to add 30+50, we can add 3 tens plus 5 tens to get 8 tens, or 80. Then students can see that 30+54 is just 4 more than 30+50.

$$30+50=80$$
3 tens + 5 tens = 8 tens

$$30+54=84$$
30+54 is 30+50+4

Finally, 39+54 is 9+4 more than 30+50. In other words, we can add the tens and ones separately, then combine them. 39+54 is 30+50 plus 9+4. That’s 80+13 = 93. This may seem like a lot of steps, but they are quick, easy, and make sense. When students understand *why* what they are doing works, they can build on what they know.



Beast Academy 1
Chapter 8: Comparing

Sequence: •••

BA1, Chapter 7	BA1, Chapter 8	BA1, Chapter 10	BA2, Chapter 1
Addition & Subtrac...	This Chapter	Big Numbers	Place Value

•••

Students should recognize and be able to use the symbols for less than (<) and greater than (>). Students should understand that when comparing two amounts, we can ignore any parts that are equal (this was a heavy focus of Chapter 3 and continues to be important in this chapter).

Overview

Students begin the chapter with a review of the comparison symbols <, >, and =, and practice ordering sets of numbers from least to greatest.

Then, students learn to compare sums and differences without adding or subtracting. Students can reason through and even act out scenarios to help them make comparisons, sometimes between unknown amounts.

This is a very early introduction to algebraic thinking.

Rather than teaching rules to be memorized, encourage students to explore how numbers work. This will help solidify their understanding of numbers and operations. For example, students can discover that adding or subtracting the same amount from two numbers doesn't change which is larger, or even the difference between them!

Comparing Sums

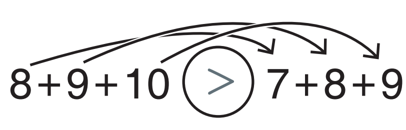
First, students are asked to compare amounts without actual numbers. This is an early introduction to algebraic thinking!

For example, if students are told that Anna is older than John, they can figure out that in 10 years Anna will still be older than John. In algebra, we write that if $A>J$, then $A+10>J+10$.

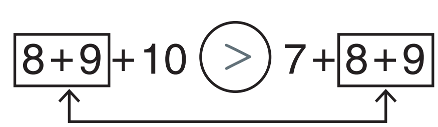
If students are told that Anna and John are the same height, and Anna grows 4 inches while John grows only 3 inches, they can figure out that Anna will then be taller. Algebraically, we say that if $A=J$, then $A+4>J+3$.

Although students at this level will not yet use variables, they are applying the same algebraic logic. Encourage students to explain their reasoning in words. "Since they started out the same height, and Anna grew more, Anna will be taller now." This builds the foundation for algebra.

Students then apply these concepts to compare sums with actual numbers, but without ever doing any addition. This usually involves one of two methods. Students either compare corresponding parts on both sides, or find the parts that are the same and ignore them.



Each number on the left is larger than the corresponding number on the right. So, the sum on the left is larger.



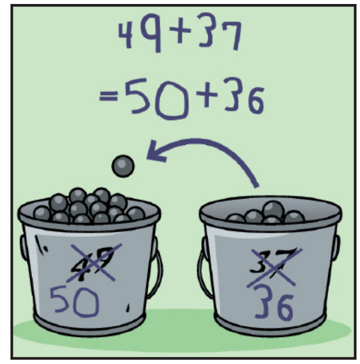
These parts are the same, but 10>7. So, the sum on the left is larger.

Both strategies let us compare without ever adding the numbers.

Beast Academy 1
Chapter 8: Comparing

Equal Sums

Next, students are asked to find sums that are equal. When adding two numbers, you can increase one number and decrease the other by the same amount without changing their sum.



Do not introduce this as a rule. After all, this is not a "rule" that mathematicians made up. It just works. Ask students to explain why. Encourage concrete models. When we move an amount between two numbers, the total amount doesn't change. If students keep this in mind, there is no need to memorize a rule. They can just apply what they already know. This is useful for making sums that are easier to compute.

Comparing Differences

As with the addition before it, students begin by comparing amounts without knowing any actual numbers. For example, if Anna and John have the same number of raisins, but Anna eats more raisins than John, then John will have more left. Encourage students to reason through these problems or even act them out. They can choose lots of different numbers and reason through them, then try to explain why the answer stays the same no matter what numbers they choose. In this example, students should see that taking away more leaves less.

Students then apply these concepts to compare differences with actual numbers. They learn ways to do this without doing any subtraction, like in the examples below.

$84 - 37 < 84 - 36$

We started with the same amount on both sides. But, we took away more on the left. So, the left side is smaller.

$85 - 36 > 84 - 36$

We started more on the left, but we took away the same amount. So, the left side still has more.

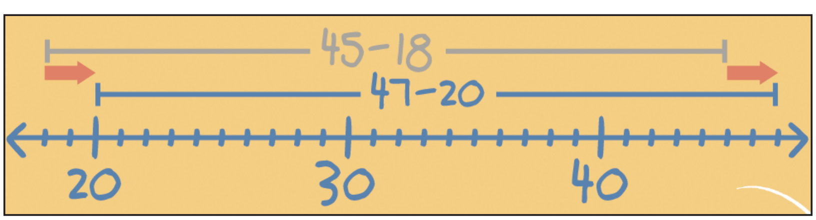
Both strategies let us compare without ever subtracting the numbers.

Equal Differences

Next, we learn to find differences that are equal. When subtracting two numbers, you can increase or decrease both numbers by the same amount without changing their difference.

Again, do not introduce this as a rule. Ask students to explain why it works. A number line model is useful here. When we move two numbers up or down the number line by the same amount, the distance between them (their difference) doesn't change.

This can help make differences that are easier to compute.



Students who try to memorize rules for equal sums or differences will easily confuse them. Students who have a mental model for why they work will have an easier time applying them correctly.

Beast Academy 1
Chapter 9: Patterns

Sequence: BA1, Chapter 9 This Chapter → BA3, Chapter 2 Skip-Counting → BA5, Chapter 7 Sequences

Students should be fluent adding and subtracting within 100 and should have some experience recognizing rotations of shapes.

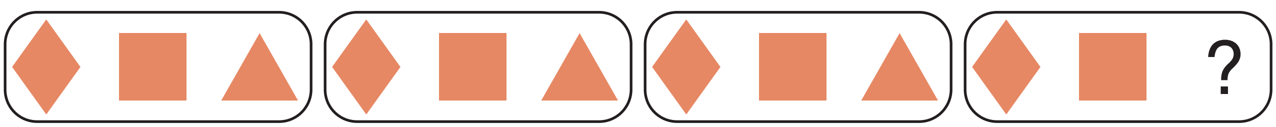
Overview

In this chapter, students work with shape patterns and begin skip-counting by the numbers from 2-9. The chapter also includes lots of enrichment material, including famous number patterns like the Fibonacci sequence and explorations of perfect squares and triangular numbers.

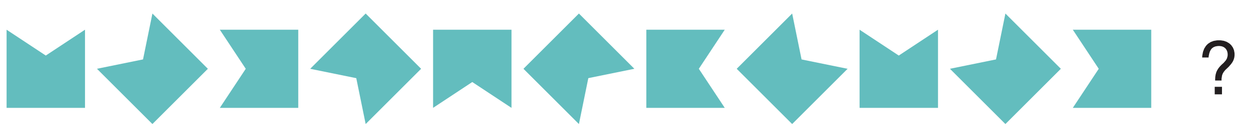
If you are short on time and need to move quickly through a chapter, the enrichment material in this chapter may be skipped.

Shape Patterns

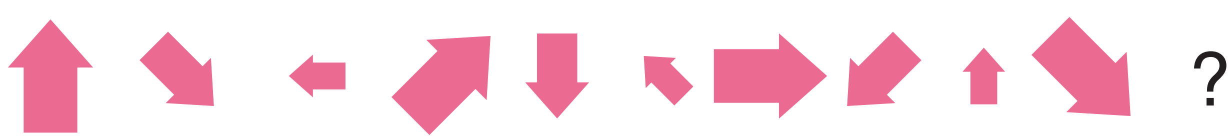
We help students recognize a variety of patterns that involve shapes. Some patterns just repeat the same set of shapes over and over again. Students should try to find the repeating part.



Other patterns involve recognizing rotations. (Next in the pattern below is .)



Some patterns are made by combining two or more patterns. For example, in the pattern below, the shape rotates while also changing size.



It can help for students to look at traits separately. Paying attention only to the size of the shape, students can notice that the arrows go from big, to medium, to small, and then repeat.

Next, they might see that the arrows go back and forth between diagonal (↘↗↖↙) and orthogonal (↑←↓→), as they rotate.

Putting these together, students can find that the next shape is a medium arrow that points left.

Practicing recognizing patterns like these will help students learn what to look for in other patterns. Young minds are great at pattern recognition, and such practice sharpens a skill that is often useful in more advanced mathematics.

Beast Academy 1
Chapter 9: Patterns

Number Patterns

Students look at two different types of number patterns in this section.

Skip-Counting

First we cover skip-counting patterns, where the same number is added (or subtracted) repeatedly.

+3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3 +3

3 6 9 12 15 18 21 24 27 30 33 36 39 42 45 48 51 54...

Students practice counting by the numbers 2 through 9 both forwards and backwards. This will help them learn their multiplication facts later on. You can start preparing students for multiplication with questions like, “What do you get if you add seven 5’s?” or “Adding ten 3’s gives 30, what do you get if you add nine 3’s?”.

The Fibonacci Sequence

In the Fibonacci sequence, each term is the sum of the two terms that come before it. Let students uncover the rule for the pattern on their own. The numbers get big fast!

1 1 2 3 5 8 13 21 34 55 89 144 233 377 610 987...

In the Practice pages, we use the same rule to start the sequence with any two numbers. This gives good addition practice.

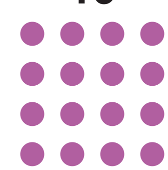
Special Patterns

We finish the chapter with a few number patterns that are related to shapes.

Perfect Squares

We first introduced perfect squares in the Categories chapter of book 1B. Here, we include them again and discover a cool connection between perfect squares and odd numbers. Every perfect square n^2 is the sum of the first n odd numbers. For example, the first 4 odd numbers sum to $1+3+5+7=16=4^2$. Find out why in the Practice pages.
(We do not use exponents or variables in the practice book.)

16



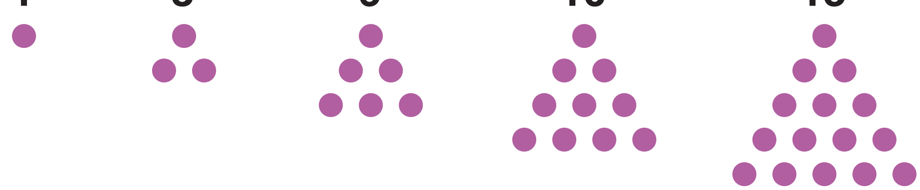
16 dots can be arranged into a 4-by-4 square, so 16 is a **perfect square**.

Triangular Numbers

While most students learn about perfect squares, many have never heard of triangular numbers. We explore a variety of problems related to triangular numbers that involve counting ways to choose two items from a group, giving a very early introduction to counting combinations.

The first 5 triangular numbers are 1, 3, 6, 10, and 15, and they continue forever. The nth triangular number can be found by adding the numbers 1 through n. For example, $1+2+3+4=10$ is the 4th triangular number.

1 3 6 10 15



Beast Academy 1
Chapter 10: Big Numbers

Sequence: ••• BA1, Chapter 8 Comparing ▸ **BA1, Chapter 10 This Chapter** ▸ BA2, Chapter 1 Place Value ▸ BA2, Chapter 2 Comparing •••

Students should be fluent counting, adding, and subtracting within 100. This includes using strategies on the number line as well as adding tens and ones separately (as taught in Chapter 7).

Overview

In this chapter, students begin working with numbers up to 999. Students practice adding small amounts to three-digit numbers (e.g. 696+8) by counting up and using number line strategies. They also practice counting up by tens to find sums like 680+50.

Students learn that a three-digit number describes a number of hundreds, tens, and ones. We introduce regrouping strategies and learn that any number ending in zero describes a number of tens (680 is 68 tens, for example). This can be very useful when adding large numbers.

Finally, students practice writing numbers in expanded form (567 = 500+60+7) and splitting numbers in other ways that can make addition easier.

This goes beyond the Common Core standards for Grade 1, but provides an important foundation for BA Level 2.

Beyond 100

Students begin by learning to read and write numbers from 100 to 999 using words and numerals.

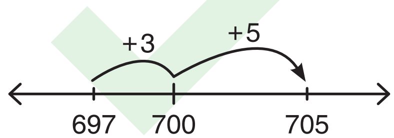
Students who are fluent counting from 0 to 99 should have an easy time counting within each hundred (from 600 to 699, for example). The difficulty often comes when adding across a new hundred. Adding 617+8 is not much more difficult than 17+8, but adding 697+8 can be tricky, especially for students who try to apply stacking algorithms for addition like the one below. (These algorithms do not appear until the end of BA Level 2, but are often taught earlier in other curricula.)

Students should apply basic strategies like counting up and using the number line.

Add 697+8.

Encourage This

"I can add 3 to get to 700, then 5 more to get 705. 697+8 is 705."



Not This

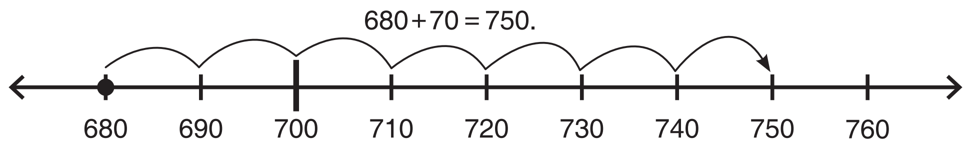
"I stack the two numbers. First, I add 7+8 = 15. I write the 5 under the 8 and write the 1 over the 9. Then I add 1+9 = 10. I write the 0 below the 9 and the 1 above the 6. Then, 1+6 = 7, so the answer is 705."

	1	11	11
697	697	697	697
+ 8	+ 8	+ 8	+ 8
	5	05	705

Beast Academy 1
Chapter 10: Big Numbers

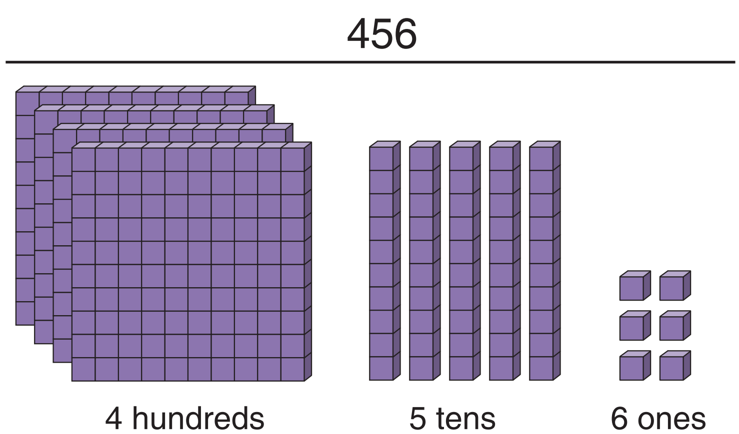
Tens

Students should get plenty of practice counting by tens beyond 100. Then, they can begin adding multiples of ten by counting up. For example, adding 680+70 by counting up by 7 tens:



Block Models

Next, we introduce a common visual model that helps students understand place value. Numbers are represented as blocks arranged into hundreds, tens, and extras (ones).



Regrouping

Using block models like the one above, students can see that 10 ones can be grouped to make 1 ten, or that 10 tens can be grouped to make 1 hundred. Similarly, we can break hundreds into tens and tens into ones. Many addition and subtraction strategies depend on students understanding breaking and regrouping.

Expanded Form

We finish the chapter by exploring the expanded form of three-digit numbers. We practice splitting and recombining numbers and their parts. For example, students should see that an amount like 687 can be separated into parts in many ways.

$$687 = 600+80+7 = 600+87 = 680+7$$

Recognizing every number as the sum of its parts will help students split large numbers into parts that are easier to add. For example, they can add 687+25 as 680+20+7+5 (since 680+20 is something they can do quickly in their heads).

Beast Academy 1
Chapter 11: Measurement

Sequence: BA1, Chapter 11 This Chapter → BA2, Chapter 7 Measurement → BA3, Chapter 9 Measurement

Students should be fluent adding and subtracting within 100.

Overview

In this chapter, students learn to compare and measure lengths and are introduced to the concept of units. The focus then shifts to measuring time in days, weeks, and months as well as minutes and hours.

This chapter has several pages where students may need help carefully cutting out shapes. You can print copies of the pages at BeastAcademy.com/Resources

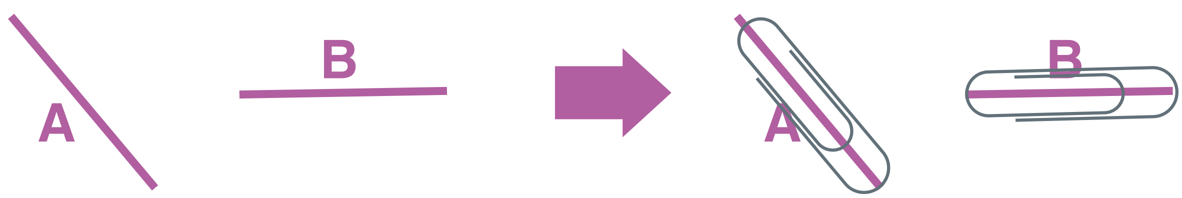
Comparing Lengths

The most straightforward way to tell whether one item is longer than another is to align them side by side. Students begin the chapter comparing side lengths of cut-out shapes directly.

Sometimes, side-by-side comparison isn't possible, so students need to find ways to compare lengths indirectly. We introduce two useful methods of indirect comparison:

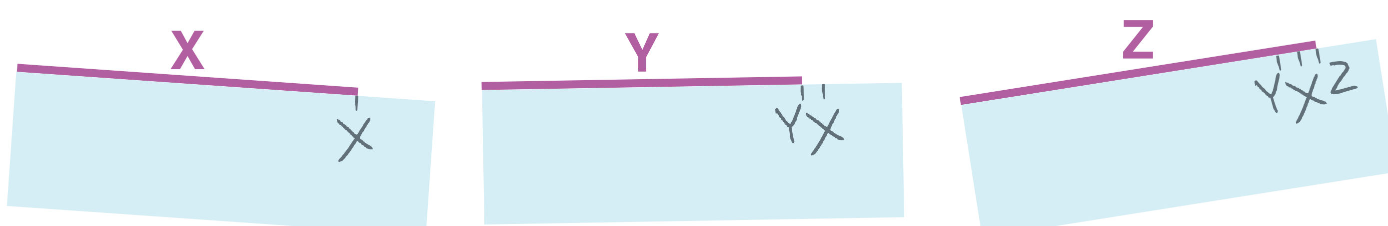
Compare items to a different object of similar length.

For example, to compare the lengths of lines A and B below, we can place a paperclip on each. Since line A is slightly longer than the paperclip, and line B is slightly shorter than the paperclip, we can see that A is longer than B.



Mark the lengths on something else.

We can mark the lengths of several items on the same piece of paper to compare their lengths. Below, we can see that line Y is shorter than X, which is shorter than Z.



Beast Academy 1
Chapter 11: Measurement

Units

Before students begin measuring with a ruler, it's important that they understand **units of measure**. Common units like inches and centimeters are abstract and tough for students to grasp quickly, so we begin by using units that students are familiar with like pennies and Lego bricks.

For example, since U.S. pennies are always the same size, we can use them to measure and compare lengths. The line below is 5 pennies long. A person anywhere in the world could draw a line exactly the same length as this one (as long as they have five pennies) because pennies are always the same size.

Rulers

Once students understand the concept of a unit, we introduce centimeters and teach students to measure objects with both a penny ruler and a centimeter ruler.

Time

Days, Weeks, and Months

Students are first introduced to days, weeks, and months. Working with days of the week gives students a very early introduction to modular arithmetic. Since the days of the week repeat every 7 days, students can see that it will be the same day of the week in 3 days as it will be in $3+7=10$ days and in $3+7+7=17$ days. We can keep going, counting up by sevens.

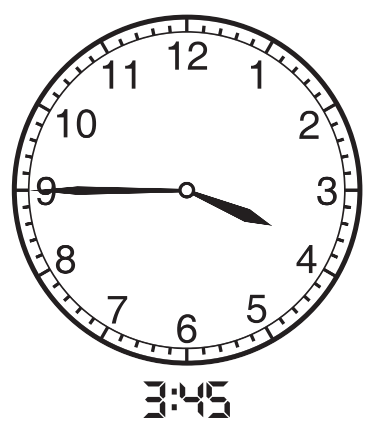
Similarly, since the months repeat, students can see that 1 month from today it will be the same month as $1+12=13$ months from today and $1+12+12=25$ months from today.

Clocks

Students learn to read time on both digital and analog clocks. The hurdle for students working with time on digital clocks usually involves crossing hours, especially from 12:59 to 1:00. Reading an analog clock presents several other challenges:

Minutes are not numbered. Students will need to count by fives to find the minutes on an analog clock. This is great practice for learning multiples of five. Students can also remember key numbers like 15, 30, and 45 and count from there.

The hour hand is always moving. The hour hand points to the 3 at 3 o'clock. Between 3:00 and 4:00, the hour hand moves slowly towards the 4. Students might look at which number the hour hand is closest to. At 3:45, though, the hour hand is much closer to the 4 than the 3. Have students guess the time without looking at the minute hand. On the right, the hour hand is close to the 4, so a reasonable guess is "Between 3:30 and 4:00."



Students might read the time above as 4:09, since the hour hand is nearest the 4 and the minute hand is pointing at the 9. The best practice come from reading real times on real analog clocks.

Beast Academy 1
Chapter 12: Problem Solving

AoPS Beast Academy
Art of Problem Solving

Sequence: BA1, Chapter 12 This Chapter → BA2, Chapter 6 Problem Solving → BA2, Chapter 9 Odds & Evens → BA2, Chapter 12 Problem Solving

Students should be fluent adding and subtracting within 100. It will also help if students know left from right and have some experience with the four cardinal directions (north, south, east, and west).

Overview

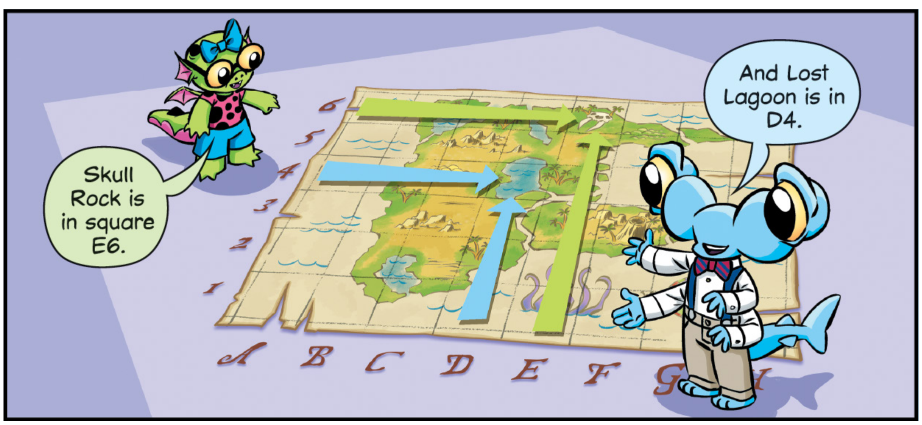
This chapter was almost named "Directions" because it is primarily a chapter about reading and understanding how to follow directions. Students practice with words like left, right, top, bottom, and middle to describe position. Students then learn to use coordinates on a grid.

Students use the cardinal directions (north, south, east, and west) as well as directions involving turns (left and right) to find their way around actual maps.

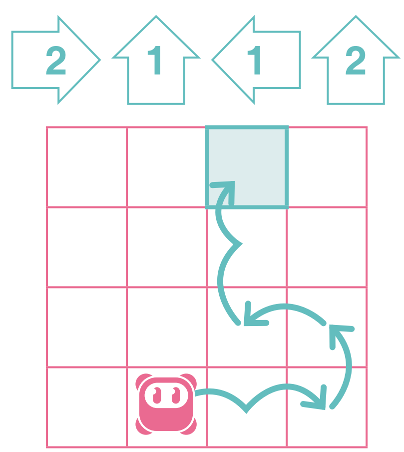
Throughout the chapter, students are asked to consider the importance of order in directions. There are times when the order of directions is important, and other times when order doesn't matter.

Position

Throughout the curriculum, students must be able to understand descriptions of positions, especially on grids. Phrases like "top-left" or "left column" or "center row" are used to describe locations in BA. We also introduce coordinates, which can be used to name squares as shown below.



Directions



Next, we introduce directions that involve movement on a grid.

Ortho Bots

The first bots we introduce move up, down, left, or right any number of squares. Students are asked to find shortcuts. The goal is for students to recognize how commands combine or cancel each other.

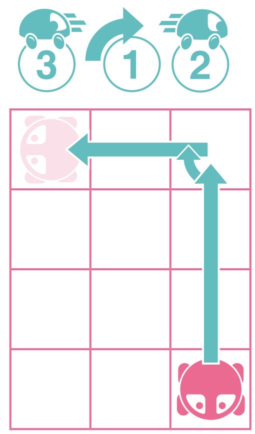
In this case, going up 1 then up 2 is the same as going up 3. Going right 2 then left 1 is the same as going right 1.

Shorter directions could be →1 ↑3 or ↑3 →1.

Directions (cont.)

Turn Bots

Next, students are introduced to Turn Bots. Turn Bots can move forward or backward, and rotate in quarter-turn increments.



Turns can be tricky since turning right or left depends on the direction the bot is facing at the time. For example, the bot on the right rotates right and ends up facing left. Students will have to consider which way the bot is facing at each step. It may even help for students to act some of these out.

An important difference between Turn Bots and Ortho bots comes at the end of each section, where we ask students whether the order of the commands changes where the bots end up.

With Ortho Bots, rearranging the commands changes the *path* a bot takes, but does not change the final destination square. Going [1→] [3↑] will lead a bot to the same square as going [3↑] [1→].

With Turn Bots, order matters. Turning right then driving forward will not take a Turn Bot to the same square as driving forward then turning right. Rearranging instructions changes the destination.

This leads us to the next section which is specifically about order.

Order

Students should consider the order of events and instructions.

For many instructions, order is important. Other times, some steps can be rearranged. For example, to build the block tower on the right, we must place E first and A last. But the order of blocks D, B, and C doesn't really matter.

When evaluating math expressions, students are taught to follow a specific order of operations. At this level, students will usually work from left-to-right to add and subtract.

But, there are many problems that can be solved more efficiently with some rearrangement. Below are three types of problems that students will encounter in Level 2 where working from left-to-right is not always best.

$49+75+51+25=\square$ $\quad$ $456-70-56=\square$ $\quad$ $394-30+31=\square$

In the first, we can pair numbers that sum to 100 to get $100+100=200$.

In the second, it's easiest to take away 56 first to get 400. Then, we subtract 70 to get 330.

In the third, students may notice that we add 1 more than we take away. So, we have $394+1=395$.

We don't introduce these specific strategies in Level 1, but we encourage students to look for efficient ways to solve problems and to think about when the order of steps matters.