

MATH TEAM

Arithmetic Sequences



One of the first things you ever learned in math was a sequence...

...when you learned to recite the positive integers.



Later on, you learned how to skip-count.

Starting at a number, we always add the same amount to get the next term.

For example, the terms of this sequence start at 50 and count by 7's.

A sequence where we always add the same amount to get from one term to the next is called an **arithmetic sequence**.

How could you find the 100th term of this arithmetic sequence?

$$50, 57, 64, 71, 78, \dots$$



What is the 100th term?



We add 7 to get from each term to the next.

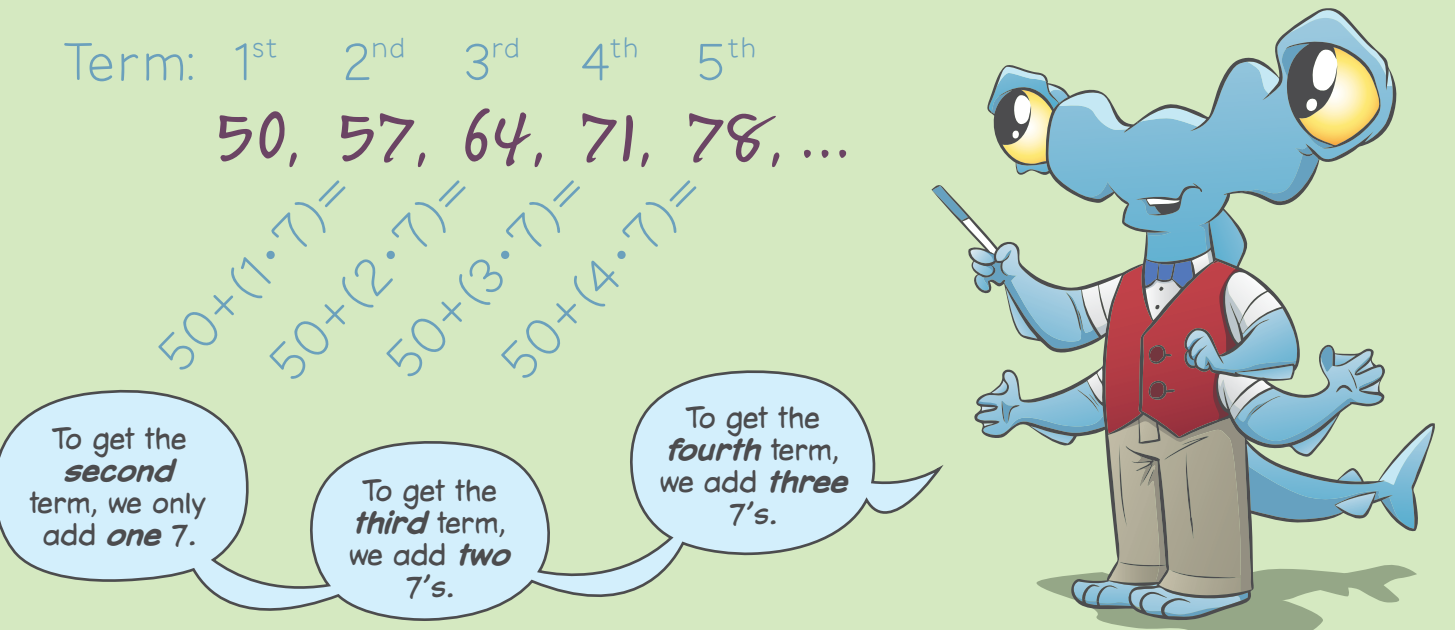
We just need to know how many 7's we add to get the 100th term.

To get the 100th term, can we just add one hundred 7's?

I don't think that's right.

$+7$ $+7$ $+7$ $+7$

50, 57, 64, 71, 78, ...



Term: 1st 2nd 3rd 4th 5th

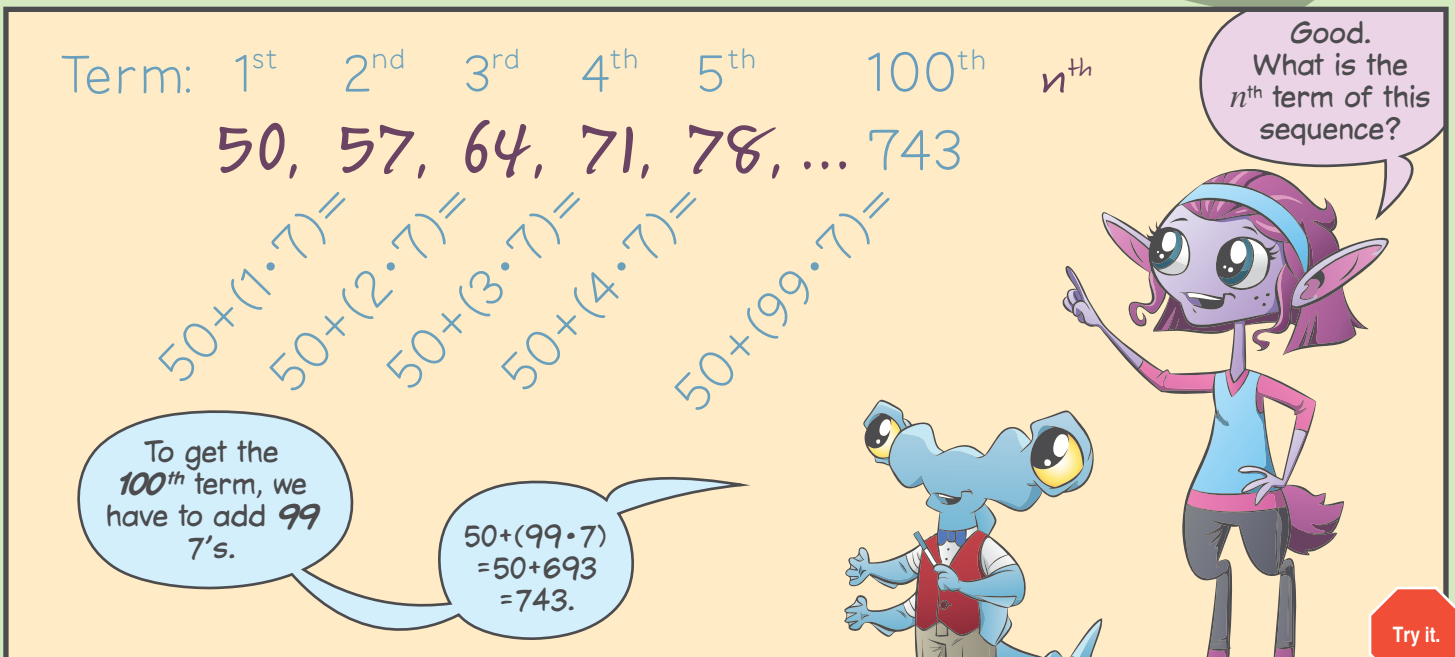
50, 57, 64, 71, 78, ...

$50 + (1 \cdot 7) =$
 $50 + (2 \cdot 7) =$
 $50 + (3 \cdot 7) =$
 $50 + (4 \cdot 7) =$

To get the **second** term, we only add **one** 7.

To get the **third** term, we add **two** 7's.

To get the **fourth** term, we add **three** 7's.



Term: 1st 2nd 3rd 4th 5th 100th n^{th}

50, 57, 64, 71, 78, ... 743

$50 + (1 \cdot 7) =$
 $50 + (2 \cdot 7) =$
 $50 + (3 \cdot 7) =$
 $50 + (4 \cdot 7) =$
 $50 + (99 \cdot 7) =$

To get the 100th term, we have to add **99** 7's.

$50 + (99 \cdot 7)$
 $= 50 + 693$
 $= 743.$

Good. What is the n^{th} term of this sequence?

Try it.

To get the n^{th} term, we have to add $(n-1)$ 7's.

Term: 1st 2nd 3rd 4th 5th 100th n^{th}
 50, 57, 64, 71, 78, ... 743 $50+7(n-1)$

$$50+(1 \cdot 7) =$$

$$50+(2 \cdot 7) =$$

$$50+(3 \cdot 7) =$$

$$50+(4 \cdot 7) =$$

$$50+(99 \cdot 7) =$$

So, the n^{th} term is $50+7(n-1)$, which simplifies to $7n+43$.*



*TO SIMPLIFY $50+7(n-1)$, DISTRIBUTE THE 7 TO GET $50+7n-7$, THEN COMBINE LIKE TERMS TO GET $7n+43$.

Well done.

+7 +7 +7 +7
 50, 57, 64, 71, 78, ...



The amount you add is called the **common difference** of the sequence.

This sequence has a common difference of 7.

Find the common difference for each of these arithmetic sequences.



199, 298, 397, 496, ...

13, 5, -3, -11, -19, ...

$\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, ...




We can find the common difference in an arithmetic sequence by subtracting any term from the next term.

The common difference is 99.

+99 +99 +99

199, 298, 397, 496, ...




In this sequence, the numbers are decreasing!

Each number is 8 less than the one before it.

We can add -8 to get the next term, so the common difference is -8.

-8 -8 -8 -8

13, 5, -3, -11, -19, ...



This one looked hard at first...


...until I changed all of the fractions into twelfths.

The common difference is $\frac{1}{12}$.

$\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, ...

$\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, $\frac{6}{12}$, ...

$+\frac{1}{12}$ $+\frac{1}{12}$ $+\frac{1}{12}$ $+\frac{1}{12}$



Perfect. Use these common differences to compute the 21st term of each sequence.

+99 +99 +99

199, 298, 397, 496, ...

-8 -8 -8 -8

13, 5, -3, -11, -19, ...

$+\frac{1}{12}$ $+\frac{1}{12}$ $+\frac{1}{12}$ $+\frac{1}{12}$

$\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{5}{12}$, $\frac{1}{2}$, ...

